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Markov Chains and Card Shuffling

In 1990, a New York Times article declared "In Shuffling Cards, 7 Is Winning Number [1]." The article reported that two mathematicians, David Aldous and Persi Diaconis [2], had published a mathematical proof that seven shuffles sufficiently mixes a deck of cards so that they appear close to truly randomized. Their results showed that the more typical three or four shuffles most people and card dealers use leave the deck far from random. The mathematical interest of this result is that card shuffling is an application where the number of steps, or shuffles, can be exactly determined that are needed to bring the Markov chain that models the shuffling close to its distribution limit. Markov chains have many applications and in almost all of them it is important to know how long the chain should be run to achieve the desired result. This project will use simulations and matrix computations to first understand the known results on various methods that have been proposed for shuffling. Then there are several shuffle models that are supposed to capture certain more realistic features of actual shuffles for which there are not a lot of mathematical results. For these shuffles, running extensive simulations could be very helpful in understanding how well they approach randomness.

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